Title: **A Space-Time Discontinuous Galerkin Method for Extended Hydrodynamics**

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The equation:

$$\partial_t \mathbf{u}(\mathbf{x}, t) + \nabla \cdot \mathbf{f}(\mathbf{u}) \quad = \tfrac{1}{\epsilon}\mathbf{s}(\mathbf{u}), \quad \mathbf{x} \in \Omega, ; t > 0$$

where: $\mathbf{u}, \mathbf{x} \in R^{3\times m}$ and $\epsilon > 0$, which is a hyperbolic PDE with stiff relaxation constants, represents a model for problems relevant to many branches of fluid mechanics. Attempts to numerically solve such systems has spawn a voluminous array of clever computational methods over the last fifty years or more. In this paper the authors have introduced and exhaustively analyzed what it seems one of the the best approaches known to the solution of such systems, at least for the 1D case. The method developed is a Discrete-Galerkin discretization, DG(1)-Hancock, that is based on Huynh's upwind moment scheme, with implicit treatment of the source term by using the fully implicit two-point Radau HA method.

In this work, the authors described the Fourier analysis of the linear hyperbolic relaxation form and then confirm the analysis by means of numerical results for the Euler equation with heat transfer. Numerical results show that the method produce highly accurate results and is computationally more efficient than semi-discrete schemes based on the Method-Of-Lines.

The paper is well written and in addition to the analysis of the numerical details of the solution method, the authors give a very good description of the variety of possible solutions based on the relative weight of the parameters in the source term. The description of several forms of the non-dimensional forms makes this work part of the list of *required reading* for CFD practitioners.

See also:

1. Suzuki, Y. *Discontinuous Galerkin Methods for Extended Hydrodynamics*, PhD Dissertation, (2008), available on-line at:
`http://deepblue.lib.umich.edu/handle/2027.42/58411?show=full`