Title: **A New Stabilized Subgrid Eddy Viscosity Method Based on Pressure Projection and Extrapolated Trapezoidal Rule for the Transient Navier-Stokes Equations**

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This paper presents the stability and convergence analysis of a new subgrid eddy viscosity algorithm for the incompressible Navier-Stokes equations:

$$\begin{aligned} \mathbf{u}_t + (\mathbf{u}\cdot\nabla)\mathbf{u} - \nu\Delta\mathbf{u} + \nabla p &= \mathbf{f}, \quad \text{in } (0,T]\times\Omega, \\ \nabla\cdot\mathbf{u} &= 0, \quad \text{in } (0,T]\times\Omega, \\ \mathbf{u} &= 0, \quad \text{in } (0,T]\times\partial\Omega, \\ \mathbf{u}(0,x) &= \mathbf{u}_0, \quad \text{in } \Omega, \\ \int_\Omega p dx &= 0, \quad \text{in } (0,T], \end{aligned} \tag{1}$$

where $\Omega \subset R^2$ is a bounded domain with boundary $\partial\Omega$, $[0,T]$ is a finite time interval, $\mathbf{u}(t,x)$ is the velocity and $p(x,t)$ is the pressure. The viscosity $\nu > 0$, the body forces $\mathbf{f}(x,t)$ and the initial velocity $\mathbf{u}_0$ are given.

The algorithm proposed by the authors is based on pressure projection and extrapolated trapezoidal rule and it uses lowest equal-order pair of finite elements. The reader interested in the current state of knowledge on the projection methods will find the very complete survey paper[1] by Guermond et al. an indispensable reference.

The scheme stabilizes convection dominated problems and ameliorates the restrictive inf-sup compatibility stability[1] by combining subgrid eddy viscosity with a pressure projection method for the spatial discretization and an extrapolated trapezoidal rule for the temporal discretization.

Some attractive features include parameter free for the pressure stabilized term and calculations required for higher order derivatives and requires only the solutions of a linear system of equations per time step and has second order temporal accuracy.

Proofs of stability and convergence are presented although the authors deferred the presentation of numerical results and the comparison with other stabilized finite element methods for a forthcoming paper.

References:

1. Guermond, J.L. et al., *An overview of projection methods for incompressible Flows* , Comput. Methods Appl. Mech. Engrg., **195**, pp 6011-6045, (2006)