Title: **Asymptotic Preserving HLL Schemes**

Author: **Berthon, C. & Turpault, R.**

Published in: *Numerical Methods for Partial Differential Equations* , **27:6**, 1396–1422, (2011)

Review by: Mario Forcinito

Conservation laws often used in physics can be modeled using the prototypical hyperbolic differential equation. When supplemented by a source term, the resulting equation can be used as a useful model for many physical phenomena. A couple of examples of these systems are the Euler equations with high friction and the M1 model for radiative transfer. Numerical solution for this type of problems are difficult because of their particular asymptotic behavior that for certain choice of parameters tends to that of the diffusion equation. This paper presents the derivation of a Godunov scheme following the classic Harten, Lax, van Leer (HLL) approach[1] to approximate the solutions of such systems that also restore the suitable asymptotic diffusive regime.

To achieve this objective, the authors introduce a free parameter into the source term, with the intent to be adjusted in such a way to satisfy the expected diffusion equation at the discrete level. The HLL scheme for hyperbolic systems with source is derived by modifying the HLL scheme for the associated homogeneous hyperbolic system.

The authors conclude that the numerical procedure presented is robust, in the sense that, by design the source term discretization preserves the physically admissible states. The scheme is illustrated with numerical experiments.

References:

1. Harten, Lax, and van Leer, *On upstream differencing and Godunov-type schemes for hyperbolic conservation laws*, SIAM Rev, **25**:1, pp 35-61, (1983).