

Title: **An hp certified reduced basis method for parametrized parabolic partial differential equations**

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The authors introduce an hp certified reduced basis (**RB**) method for parabolic partial differential equations that invokes a Proper Orthogonal Decomposition (POD)(in time)/Greedy (in parameter) sampling procedure first in the initial partition of the parameter domain (h -refinement) and subsequently in the construction of **RB** approximation spaces restricted to each parameter subdomain (p -refinement).

The certified reduced basis method for the solution of partial differential equations represents a model-order reduction framework that allows the rapid evaluation of functional outputs for PDE's that depend on an input parameter vector. This vector can be related to geometric or material property factors.

The four key ingredients to the certified RB framework are:

- Galerkin projection;
- POD/Greedy sampling;
- a posteriori error estimation; and
- OfflineOnline computational decomposition.

In this paper the authors extend their work previously introduced in [1] in several important ways by introducing an improvement to the algorithm and a priori convergence theory for the initial subdivision process, presenting a very detailed account of the algorithm and the results.

All in all the authors show how the confluence of ideas such as the h and the p refinements the use of POD and the offline/online splitting of

the computational effort result on a fast computational approach for the evaluation of functionals based on the solution of PDE's. These type of approach will certainly find a lot of practical and theoretical applications in science and engineering.

It is a remarkable fact that, almost a century after B. G. Galerkin published his method, the mathematics and engineering communities are still taking advantage of it and finding new ways to apply his ideas to the approximated solution of differential equations.

References:

1. J.L. Eftang, A.T. Patera, and E.M. Rønquist, An hp certified reduced basis method for parametrized parabolic partial differential equations, in Spectral and High Order Methods for Partial Differential Equations, Lecture Notes in Computational Science and Engineering, **76**, J.S. Hesthaven and E.M. Rønquist, eds., Springer, Trondheim, Norway (2009), pp. 179-187.

See also:

1. Dr. Eftang's Doctoral Thesis on the subject available on-line at:
<http://ntnu.diva-portal.org/smash/get/diva2:414290/FULLTEXT03>
2. Dr. Løvgren's Doctoral Thesis on the subject available on-line at:
<http://ntnu.diva-portal.org/smash/get/diva2:124326/FULLTEXT01>
3. Quarteroni, A., et. al. **Certified reduced basis approximation for parametrized partial differential equations and applications** *Journal of Mathematics in Industry* (2011) 1:3 available on-line at:
<http://www.mathematicsinindustry.com/content/pdf/2190-5983-1-3.pdf>