Title: **Stable Computing with an Enhanced Physics Based Scheme for the 3D Navier-Stokes Equations**

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Velocity-vorticity formulations for the Navier-Stokes equations proved to be very useful in the early days of CFD (see [1]) for the treatment of complex problems. It is believed that the more physically correct a numerical scheme is, the more accurate its predictions will be, especially over long time intervals. In the 1960s Arakawa's work brought to the fore the advantages that numerical methods based on tracking the evolution of quantities such as the helicity had for the study of the Navier-Stokes equations. Advances in this area are still being made by researchers working on the helicity/ensthrophy aspects of the formulations (see one of my previous reviews [2]).

In this work, the authors present some interesting enhancements to the numerical schemes for the Navier-Stokes Equations by building numerical schemes on a formulation that explicitly preserve energy and helicity, in both, the continuum and the discrete versions of the equations. Such scheme is referred to as enhanced-physics based.

The work presented here build upon previous paper from one of the authors [3] by extending the numerical scheme to non-periodic problems. As always is the case the inclusion of homogeneous Dirichlet boundary conditions represents a challenging task, requiring the use of grad-div type stabilizations.

An analysis of the error, the convergency and remarkable examples of how much better these type of schemes perform with respect to standard schemes are presented in full detail.

References

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http://www.surengineering.com/AMS/MR2594820.pdf

3. Rebholz, L. *An Energy and Helicity Conserving Finite Element Scheme for the Navier-Stokes Equations*, 2006.
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See Also

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