Title: **Adaptive Timestep Control for Nonstationary Solutions of the Euler Equations.**

Author: **Steiner, C., Siegfried Müller, S. and Noelle, S.**

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Review by: Mario Forcinito

The governing equations for the non-stationary flow of an inviscid, compressible fluid can be expressed as a system of time dependant hyperbolic conservation laws. This system of equations is known as Euler equations[1] and its numerical solution have many applications of practical interest for scientist and engineers.

A casual browsing through the literature on this subject will show, just by looking at the number of researchers and the heaps of papers devoted to the quest for efficient computational solutions for the Euler equations, its enormous relevance.

Euler equations develop singularities over finite time intervals, therefore it is convenient to base the numerical schemes on a weak form of the equations. This naturally leads to finite volume schemes as the first choice for the spatial discretization of the equations. To capture the details of the solution around the singularities, many strategies have been developed for the efficient refinement of the computational grid, involving the splitting of elements or the adaptive tracking of the singularities. For non-stationary cases the refinement of the mesh impose serious limitations to the practical computing of solutions given that the CFL limit in the time step forces a large amount of computations to advance the solution through time.

In this (and precedent) work, the authors have been mainly concerned with developing adaptive strategies for the optimal discretization in time. The fundamental idea behind their approach is to consider the finite volume, backwards Euler scheme as a particular case of the Galerkin method and then find a projection for the error functional such that the contributions of time and space discretizations can be estimated independently.

[1] A succinct presentation of the basic mathematics about Euler equations can be found here: `http://irfu.cea.fr/Projets/COAST/amr_lecture1.pdf`

The paper gives a comprehensive account of the mathematical constructs needed for the application to the Euler equations with particular care on how to treat the boundary conditions.

Among the contributions of this work the authors established a complete error representation for nonlinear initial boundary value problems with characteristic boundary conditions for hyperbolic systems of conservation laws.

The authors successfully combine a multiresolution-based grid adaptation with adjoint techniques and apply it to the efficient solution of mildly nonstationary problems. Examples of the application of these techniques to 2-D unsteady flow problems are described in detail and the gains in computational efficiency amply demonstrated.