Title: **Finite volume schemes for locally constrained conservation laws**

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Some problems of practical importance pose challenges that pushes the limits of the current understanding of differential equation solution methods. Such is the case of modeling of road traffic and how it is affected by control systems such as traffic lights or toll gates.

In this case, the differential equation can be derived from a standard conservation law, because the boundary condition is given by a local unilateral constraint on the flux, the handling of the boundary condition represents a difficult problem for the numerical solution of the problem.

The main difficulty with the boundary condition resides in the existence of a phase change for the flux function. This phase change is, in this case, related to the radically different behaviour drivers have between the situations in which the road is open or congested, which result in a boundary condition dependant on a function that has no continuous support.

In this paper the problem is interpreted in term of a conservation laws with a discontinuous flux function. The authors reformulate, accordingly, the notion of entropy solution and extend the wellposedness to the $\mathbf{L}^\infty$ framework.

The authors present the numerical simulation of a system with 5 traffic lights for which they determine the optimal delay between red lights consistent with a "green wave".

See also:

1. Colombo, R., Goatin, P. and Priuli, S., *Nonlinear Analysis 66 (2007) 24132426*, available on-line at:
   `http://goatin.univ-tln.fr/articoli/A4.pdf`

2. Colombo, R., Goatin, P. and Rosini, M., *Communications to SIMAI Congress,Vol. 3 (2009)*, available on-line at:
http://goatin.univ-tln.fr/articoli/P2.pdf