Title: **Note on helicity balance of the Galerkin method for the 3D NavierStokes equations**

Author: **Olshanskii, M. and Rebholz, L.G.**

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Review by: Mario Forcinito

In the computational solution of differential equations, it is desirable to count with schemes that emulate as many *physical* characteristics of the differential equation as possible. This is typically achieved by creating numerical schemes that emulate the conservation laws implied in the original equation system. Thus, for example, in the absence of dissipation and external forces, the Navier Stokes equations imply conservation of certain integral quantities such as the energy and the helicity. Helicity is defined in three-dimensional space as: $H = \int_\Omega u \cdot (\nabla \times u)$, where Ω is the volume of interest and u is the velocity field. In two-dimensional flows this reduces to the quantity know as Enstrophy $Ens = \frac{1}{2}\int_\Omega |\nabla \times u|^2$. These quantities are very relevant in applications such as atmospheric flows and magnetohydrodynamics.

In this paper the authors show that the standard Galerkin method preserves a slightly altered discrete helicity type quantity. Following in the observation that sometimes small modifications of known methods increase physical accuracy without a significant increase in computational work, the paper show that the usual Galerkin method with explicitly skew-symmetrized nonlinear term accurately balances both energy and a discrete helicity for the case of periodic boundary conditions (in the opinion of the authors, for non-periodic boundary conditions the problem is still open).

This work certainly is a valuable addition to our understanding of the interplay between the physical conservation laws and their Galerkin method counterpart, and it is bound to be the base for more accurate numerical schemes.

See also:

1. Rebholz, L. *An Energy and Helicity Conserving Finite Element Scheme for the Navier-Stokes Equations*, 2006.
   http://citeseerx.ist.psu.edu/viewdoc/download?doi=10.1.1.140.1056&rep=rep1&type=pdf

2. Bekenstein, J. D., *Helicity conservation laws for fluids and plasmas*, Astrophysical Journal, Part 1 (**319**), 1987, p. 207-214. available on-line at:
   http://articles.adsabs.harvard.edu//full/1987ApJ...319..207B/0000207.000.html