Title: **Third order TVD scheme for hyperbolic conservation laws**

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The author presents and analyzes a third order difference scheme for hyperbolic conservation laws. Based on the Lax-Wendroff difference scheme and the Crank-Nicholson implicit scheme, the resulting $\mathcal{O}(\Delta t^2, \Delta x^3)$ is remarkably simple and efficient. For the simple first order hyperbolic conservation law $u_t + f(u)_x = 0$; with initial conditions:

$u(x,0) = u_o(x),\ u(b,t) = g(t)\ t > 0.$

The scheme is proven stable for Courant numbers $|c| \leq \sqrt{2}$. Stability and order accuracy is proved in general for linear boundary value and initial value problems and numerical examples are provided showing the increased efficiency for with respect to the Lax-Wendroff scheme and the third order scheme by Shi and Toro[1].

To address the problem of spurious oscillation behind the wave fronts, the scheme is reformulated into a TVD method. The third order TVD scheme is then applied to linear and nonlinear systems. Examples are presented for the one-dimensional shallow waters equations (linearized and nonlinear forms) as well as a model two dimensional problems.

1. J. Shi and E. F. Toro , *Fully discrete high order shock capturing numerical schemes*, International Journal for Numerical Methods in Fluids, vol. 23, pp. 241–269, 1996.

Related:

- V. A. Titarev, E. F. Toro , *MUSTA schemes for multi-dimensional hyperbolic systems: analysis and improvements*, International Journal for Numerical Methods in Fluids, vol. 49, no. 2, pp. 117-147, 2005.

- E.F. Toro and V.A. Titarev, *MUSTA schemes for systems of conservation laws*, Journal of Computational Physics, vol. 216, no. 2, pp. 403-429, 2006.

- Y. H. Zahran, *A Central WENO-TVD Scheme for Hyperbolic Conservation Laws*, Novi Sad J. Math., vol. 36, no. 2, pp. 25-42, 2006.

- Y. H. Zahran *MUSTA-TVD Scheme for Hyperbolic Conservation Laws*, Comptes rendus de l'Acadmie bulgare des Science, Tome 59, no. 9, pp. 911–920, 2006.